11-1 Modeling

Point!

Modeling

(1) ([1]**Model**) : A simplified representation of a target phenomenon.

(2) ([2]**Modeling**) : The process of creating models for events or phenomena.

(3) Classification of Models by Characteristics

[1] ([3]**Static model**) : A model that does not undergo changes over time.
<Examples> Room layout diagrams, the relationship between the radius and volume of a sphere, etc.

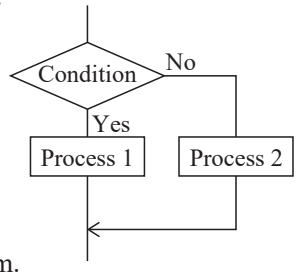
[2] ([4]**Dynamic model**) : A model that is influenced by changes over time.

- ([5]**Deterministic model**) : A model that does not exhibit irregular behavior. There is only one result.
<Examples> Bank account balance, relationship between time and travel distance for a car moving at a constant speed, etc.
- ([6]**Probabilistic model**) : A model that includes irregular behavior. The result is not a single outcome.
<Examples> Outcomes of dice rolls, lottery draws, weather forecasts, etc.

(4) Classification of Models Based on Type of Expression

[1] ([7]**Physical model** (**scale model**)) : A physical representation of an object.

- ([8]**Enlarged model**) : A model that is larger than the actual object.
<Examples> Molecular model, DNA model, etc.
- ([9]**Full-scale model**) : A model that is the same size as the actual object.
<Examples> Model room, life-size dummy, etc.
- ([10]**Scale model**) : A model smaller than the actual object.
<Examples> Globe, plastic model, etc.



Flowchart

[2] ([11]**Diagrammatic model**) : A representation of something using a diagram.
<Examples> Flowcharts, route maps, etc.

[3] ([12]**Mathematical model**) : A representation of something using mathematical formulas or logical expressions.
<Examples> Speed × Time = Distance,
Current × Resistance = Voltage, etc.

(5) Modeling steps

[1] Clearly define the ([13]**purpose**) of modeling.

[2] Clarify the ([14]**elements that constitute the model**) and their relationships.

[3] Determine the ([15]**representation method**) of the model.

Warm Up

Answer the following questions.

(1) Choose the classification of the models most related to sentences [1] to [4] from the options **A** to **C** below, and answer using the letters.

[1] The relationship between the radius and area of a circle

[2] Outcome of a dice roll

[3] Weather forecast

[4] The relationship between time and distance traveled for a bicycle at a constant speed

A Static model **B** Deterministic model **C** Probabilistic model

(2) Choose the classification of the models most related to sentences [1] to [4] from the options **A** to **C** below, and answer using the letters.

[1] Molecular model [2] Newton's equation of motion

[3] Flowchart of a program [4] Plastic model

A Physical model **B** Diagrammatic model **C** Mathematical model

Explanation

(1) [1] **A** [2] **C** [3] **C** [4] **B**

(2) [1] **A** [2] **C** [3] **B** [4] **A**

Try

Answer the following questions.

(1) Complete the following sentences by filling in the blanks [1] to [5] with the appropriate terms.

> ([1]) are referred to as a simplified representation of a target phenomenon. Types of ([1]) include: a ([2]) model, which is a physical representation of an object; a ([3]) model, which uses mathematical expressions or logical formulas to represent the behavior of phenomena; and a ([4]) model, which uses diagrams to represent phenomena. Examples of ([2]) models include full-scale models, ([5]) models, and scale models.

(2) Choose the classification of the models most related to sentences [1] to [4] from the options **A** to **C** below, and answer using the letters.

[1] A single die was rolled once to calculate the probability of rolling a five.
[2] A layout diagram of a room was created to rearrange its furnishings.
[3] The interest after five years was calculated with an interest rate of 1%.
[4] A prediction of the weather tomorrow.

A Static model **B** Deterministic model **C** Probabilistic model

(3) Choose the models that correspond to [1] to [3] from the options **A** to **F** below, and answer using the letters.

[1] Enlarged model [2] Full-scale model [3] Scale model

A Model room **B** Globe **C** Life-size dummy
D Plastic model **E** DNA model **F** Molecular model

Exercise

1 Cover the **Point!** section on page 141 with a red sheet and test yourself by writing the items in order in your notebook.

2 Answer the following questions.

(1) Choose the term that best fits into the blanks [1] to [3] in the following sentence from the options **A** to **D** below, and answer using the letters.

> The process of creating models of events or phenomena is referred to as ([1]). Additionally, models can be broadly classified based on their characteristics into two categories: ([2]) models that do not undergo changes over time, and ([3]) models that are influenced by changes over time.

A Physical **B** Static **C** Dynamic **D** Modeling

(2) Choose the models that correspond to [1] to [3] from the options **A** to **F** below, and answer using the letters.

[1] Physical model [2] Diagrammatic model [3] Mathematical model

A Plastic model **B** Flowchart **C** Life-size dummy
D Route map **E** DNA model **F** Ohm's law

11-2 Simulations [1]

Point!

1 Methods of Referencing in Spreadsheet Software

(1) ([1]**Relative reference**) : A method of referencing where the position of the referenced cell changes automatically.

(2) ([2]**Absolute reference**) : A method of referencing that keeps the position of the row or column of the referred cell consistently fixed.
By adding "([3]**$**)," it becomes an absolute reference.
<Example> If you copy the formula as "C3," every formula will be able to refer to cell C3.

2 Simulations

(1) ([4]**Simulation**) : Manipulating a model in order to predict phenomena or events. Even when it is difficult to use actual objects, predictions can be made through simulations.

(2) Simulations Using Spreadsheet Software

[1] Water volume simulation

<Example> The relationship between the time water is poured into an empty container and the amount of water accumulated in the container

[Formula] Water volume = ([5]**(Water inflow rate) × (Elapsed time)**)

	A	B	C	D
1	Simulation of water inflow rate			
2	Water inflow rate [L/min]		Elapsed time [minutes]	Water volume [L]
3	6		0	0
4			1	6
5			2	12
6			3	18
7			4	24

	A	B	C	D
1	Simulation of water inflow rate			
2	Water inflow rate [L/min]		Elapsed time [minutes]	Water volume [L]
3	6		0	= A3*C3
4			1	= A3*C4
5			2	= A3*C5
6			3	= A3*C6
7			4	= A3*C7

[2] Simulation of balance

・ ([6]**Compound interest method**) : A method in which the interest generated from the principal (balance) is incorporated into the principal for the next period.

[Formula] Interest = ([7]**(Current balance) × (Interest rate)**)
Next period's balance = ([8]**(Current balance) + (Interest)**)

	A	B	C	D	E	F
1	Simulation of account balance					
2	Principal amount (in yen)	Interest rate (%)		Number of years	Account balance (in yen)	Interest (in yen)
3	1,000,000	3%		0	1,000,000	0
4				1	1,030,000	30,000
5				2	1,060,900	30,900
6				3	1,092,727	31,827

	A	B	C	D	E	F
1	Simulation of account balance					
2	Principal amount (in yen)	Interest rate (%)		Number of years	Account balance (in yen)	Interest (in yen)
3	1,000,000	3%		0	= A3	0
4				1	= E3 + F4	= E3*B3
5				2	= E4 + F5	= E4*B3
6				3	= E5 + F6	= E5*B3
7						

Warm Up

Initially, 1,000 yen was saved. Using spreadsheet software, a table was created like the one below to simulate the relationship between the number of months and the amount saved when saving 500 yen each month. Answer the following questions.

	A	B	C	D	E
1	**Allowance simulation**				
2	**Initial savings amount (in yen)**	**Allowance (yen)**		**Number of months**	**Savings amount**
3	1,000	500		0	1,000
4				1	1,500
5				2	2,000
6				3	2,500

(1) Can this model be represented as a deterministic model or a probabilistic model?

(2) Choose the term that best fits into the blanks [1] to [3] in the following formula from the options **A** to **D** below, and answer using the letters.

Savings amount = [1] + [2] × [3]

A Number of months **B** Allowance

C Initial savings amount **D** Amount saved last month

(3) The formula entered in cell E4 in the diagram is as follows. Note that the formula in cell E4 is copied and used to E6. Complete the following formula by filling in the blanks [1] to [3] with the appropriate answers.

[Cell E4] = [1] + [2] * [3]

Explanation

(1) The number of months and the savings amount change in a regular pattern. Therefore, it is a deterministic model.

(2) Savings in the first month = 1,000 + 500 × 1

Savings in the second month = 1,000 + 500 × 2

⋮

Therefore, [1] **C** [2] **B** [3] **A** (note: [2] and [3] can be in any order)

(3) Based on (2), enter the cell representing the initial savings amount in [1], the allowance in [2], and the number of months in [3]. Since the same cell is always used for [1] and [2], use "$" to make it an absolute reference. Also, for [3], the cell in column E is filled with the value from column D in the same row, so for cell E4, insert the value from D4. Therefore, [1] A3 [2] B3 [3] D4

Try

1 Using spreadsheet software, a table was created like the one below to simulate the relationship between the inflow time and volume of water in a container when water is poured into an empty container. Answer the following questions.

	A	B	C	D
1	**Water volume simulation**			
2	**Water inflow rate [L/min]**		**Elapsed time [minutes]**	**Water volume [L]**
3	5		0	0
4			1	5
5			2	10
6			3	15
7			4	20

(1) Can this model be represented as a deterministic model or a probabilistic model?

(2) The relationship among water volume, inflow rate, and elapsed time is expressed as follows. Complete the following formula by filling in the blanks **A** and **B** with the appropriate terms.

(Water volume) = **A** × **B**

(3) The formula entered in cell D3 in the diagram is as follows. Note that the formula in cell D3 is copied and used to D7. Complete the following formula by filling in the blanks **A** and **B** with the appropriate answers.

[Cell D3] = **A** * **B**

2 Using spreadsheet software, a table was created like the one below to simulate how the remaining balance changes when 1,000,000 yen is deposited with a bank. Answer the following questions. Furthermore, the interest generated from the principal (balance) is incorporated into the principal for the following period.

	A	B	C	D	E	F
1	**Simulation of account balance**					
2	**Principal amount (in yen)**	**Interest rate (%)**		**Number of years**	**Account balance (in yen)**	**Interest (in yen)**
3	1,000,000	4%		0	1,000,000	0
4				1	1,040,000	40,000
5				2	1,081,600	41,600
6				3	1,124,864	43,264
7						

(1) Can this model be represented as a deterministic model or a probabilistic model?

(2) What is the term for the method where interest generated from the principal (balance) is incorporated into the principal for the next period?

(3) Complete the following formulas by filling in the blanks **A** and **B** with the appropriate terms.

Interest = (Current balance) × **A**

Next period's balance = (Current balance) + **B**

(4) The formulas below are entered in cell E4 and cell F4 in the diagram. Complete the following formulas by filling in the blanks **A** to **C** with the appropriate answers. Additionally, the formula in cell E4 is copied and used to E6, and the formula in cell F4 is copied and used to F6.

[Cell E4] = E3 + **A**

[Cell F4] = **B** * **C**

Exercise

1 Cover the **Point!** section on page 144 with a red sheet and test yourself by writing the items in order in your notebook.

2 Answer the following questions.

(1) In spreadsheet software, what are the terms for the following operations?

[1] A method of referencing that keeps the position of the row or column of the referenced cell consistently fixed.

[2] A method of referencing where the position of the referenced cell changes automatically.

(2) In spreadsheet software, what symbol is used to make a formula always refer to the same cell, even when the formula is copied? Answer with the appropriate symbol.

3 Using an appliance with a power consumption of 0.5 kW means that 0.5 kWh of electricity is used each hour. Using spreadsheet software, a table was created like the one below to simulate the relationship between usage time and electricity charges when the electricity cost is 10 yen/kWh. Answer the following questions.

	A	B	C	D	E ▼
1	**Simulation of electricity bills**				
2	**Power consumption (kW)**	**Unit price (yen/kWh)**		**Usage time (h)**	**Electricity rate (in yen)**
3	0.5	10		0	0
4				1	5
5				2	10
6				3	15
7				4	20
8					

(1) Can this model be represented as a deterministic model or a probabilistic model?

(2) The relationship between electricity rate, usage time, unit price, and power consumption can be expressed as follows. Complete the following formula by filling in the blanks **A** to **C** with the appropriate terms.

(Electricity rate) = **A** × **B** × **C**

(3) The formula entered in cell E4 in the diagram is as follows. Note that cell E3 is copied and used to E7. Complete the following formula by filling in the blanks **A** to **C** with the appropriate answers.

[Cell E4] = **A** * **B** * **C**

11-3 Simulations [2]

Point!

1 Using Functions in Spreadsheet Software

(1) ([1]**Random number**) : A number that can appear with equal probability within a certain range.

(2) Functions of Spreadsheet Software

[1] ([2]**SUM (range1:range2)**) : Calculates the sum for cell range1 to range2.

[2] ([3]**IF (logical_expression, value_if_true, value_if_false)**) : When the logical expression is true, the value for "value_if_true" is displayed. When it is not true, the value for "value_if_false" is displayed.

<Example> Display "Pass" if the score is 70 or above, otherwise display "Fail"

Pass/Fail judgment
[Cell B2] =If(A2>=70, "Pass", "Fail")

	A	B
1	**Score**	**Pass/Fail judgement**
2	80	Pass
3		

[3] ([4]**=RAND()**) : Generates a random number between 0 and 1.

2 Simulation of Probabilistic Models

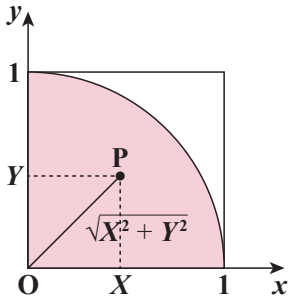
(1) ([5]**Monte Carlo method**) : A method for solving problems by using random numbers in probabilistic models.

(2) Simulation of the Approximate Value of Pi (π)

[1] Use the RAND function to randomly generate 100 points ($= N$) where both the X and Y coordinates are between 0 and 1.

[2] Count the number ($= n$) of ([6]**points that lie within a quadrant**).

[3] N: $n \approx 1$: $\frac{\pi}{4}$ Therefore, $\pi \approx \frac{4n}{N}$.



Points generated randomly using random numbers
[Cell D3] =B3^2+C3^2

Approximate value of pi
[Cell G3] =G3*4

	A	B	C	D	E	F	G	H
1	Simulation of approximate calculation of pi							
2	**Number of times**	**X-coordinate**	**Y-coordinate**	**The value of X^2+Y^2**	**Inside-outside determination**		**Probability of being inside a circle**	**Pi**
3	1	0.9066	0.9304	1.6876	0		0.800	3.200
4	2	0.2029	0.2804	0.3461	1			
5	3	0.2695	0.3481	0.4402	1			
6	4	0.7563	0.9880	1.2443	0			
7	5	0.5256	0.3030	0.6067	1			
8	6	0.1675	0.9127	0.9279	1			
9	7	0.4017	0.9968	1.0758	0			

Probability of a point being inside a circle
[Cell G3] =SUM(E3:E102)/100

Points generated randomly using random numbers
[Cell B3] =RAND()

Determines if a point is inside or outside a circle
[Cell E3] =IF(D3<=1,1,0)

*The accuracy of the approximation of the value of pi ([7]**increases**) as the number of points placed within the square increases.

Warm Up

Using spreadsheet software, as shown in the diagram below, 100 points were plotted using random numbers between 0 and 1 for the X and Y coordinates. The number of points that fall within the area of a quadrant of a circle with a radius of 1 was examined, and from this, an approximate value of pi was calculated. Answer the following questions.

	A	B	C	D	E	F	G	H
1	**Simulation of approximate calculation of pi**							
2	**Number of times**	**X-coordinate**	**Y-coordinate**	**Distance from the origin**	**Inside-outside determination**		**Probability of being inside a circle**	**Pi**
3	1	0.9090	0.8882	1.6152	0		0.800	3.200
4	2	0.6769	0.1794	0.7003	1			
5	3	0.3127	0.9045	0.9570	1			
6	4	0.4324	0.0900	0.4417	1			
7	5	0.3476	0.8694	0.9363	1			
8	6	0.7042	0.9714	1.1998	0			
9	7	0.1034	0.0875	0.1354	1			

(1) Can this model be represented as a deterministic model or a probabilistic model?

(2) What is the term for the method that solves problems using random numbers in this way?

(3) The following formulas are entered in cells B3, E3, G3, and H3 in the diagram. Complete the following formulas by filling in the blanks **A** to **D** with the appropriate answers.

[Cell B3] = **A** ()

[Cell E3] = **B** (D3<=1, 1, 0)

[Cell G3] = SUM(E3:E102)/ **C**

[Cell H3] = **D** *4

(4) When 1,000 points were randomly plotted within a square, 750 of them fell inside a quadrant. Calculate the approximate value of pi in this situation.

Explanation

(1) Probabilistic model

(2) Monte Carlo method

(3) **A**: RAND **B**: IF **C**: 100 **D**: G3

(4) Since there are 1,000 points in the entire square and 750 of them are inside the quadrant, $\frac{750}{1{,}000} \times 4 = 3$
Therefore, the value of pi is 3

Try

1 A circle with a radius of 1 is drawn within a square with a side length of 2, with the circle centered at the origin. Random points were plotted within this square, and the number of points that fell inside the circle was counted. From this, the value of pi (π) was estimated. Answer the following questions.

(1) What is the term for the method that solves problems using random numbers?

(2) The procedure for calculating pi (π) based on (1) is as follows. Complete the following sentences by filling in the blanks **A** and **B** with the appropriate terms.

> [1] Place points randomly within a square.
> [2] Determine whether those points are (**A**) the circle.
> [3] Determine the (**B**) of the area of the circle to the area of the square using the number of points inside the circle and the total number of points within the square.
> [4] Use this (**B**) to calculate the value of pi (π).

(3) How does the accuracy of the estimated value of pi change as the number of points plotted within the square increases?

(4) When 10,000 points were randomly plotted inside a square, 8,000 of them were found to be inside a quadrant. Calculate the approximate value of pi in this situation.

2 Using spreadsheet software, as shown in the diagram below, 1,000 points were plotted using random numbers between 0 and 1 for the X and Y coordinates. The number of points that fall within the area of a quadrant of a circle with a radius of 1 was examined, and from this, an approximate value of pi was calculated. Answer the following questions.

	A	B	C	D	E	F	G	H
1	**Simulation of approximate calculation of pi**							
2	**Number of times**	**X-coordinate**	**Y-coordinate**	**Distance from the origin**	**Inside-outside determination**		**Probability of being inside a circle**	**Pi**
3	1	0.9090	0.8882	1.6152	0		0.800	3.200
4	2	0.6769	0.1794	0.7003	1			
5	3	0.3127	0.9045	0.9570	1			
6	4	0.4324	0.0900	0.4417	1			
7	5	0.3476	0.8694	0.9363	1			
8	6	0.7042	0.9714	1.1998	0			
9	7	0.1034	0.0875	0.1354	1			

(1) What is the term for the method that solves problems using random numbers in this way?

(2) The following formulas are entered in cells B3, E3, G3, and H3 in the diagram. Choose the formula that best fits into the blanks [1] to [4] from the options **A** to **D** below, and answer using the letters.

[Cell B3] = [1]

[Cell E3] = [2]

[Cell G3] = [3]

[Cell H3] = [4]

A G3*4 **B** IF(D3<=1, 1, 0)

C SUM(E3:E10002)/1000 **D** RAND()

Exercise

1 Cover the **Point!** section on page 148 with a red sheet and test yourself by writing the items in order in your notebook.

2 Answer the following questions.

(1) Regarding the IF function of spreadsheet software, choose the appropriate combination of terms to fill in blanks [1] to [3] from the options **A** to **D** below, and answer using the letter.

IF(([1]), ([2]), ([3]))

A [1] logical_expression [2] value_if_true [3] value_if_false

B [1] logical_expression [2] value_if_false [3] value_if_true

C [1] value_if_true [2] value_if_false [3] logical_expression

D [1] value_if_false [2] value_if_true [3] logical_expression

(2) Using spreadsheet software as shown in the figure on the right, you want to display a 1 in cell B3 if the score in cell A3 is 60 or above, and a 0 if it is not. Choose which function should be entered in cell B3 from the options **A** to **D** below, and answer using the letter.

	A	B
1	**Pass/Fail judgment**	
2	**Score**	**Pass/Fail judgement**
3	80	1

A [Cell B3] =IF(A3<=60, 1, 0) **B** [Cell B3] =IF(A3<=60, 0, 1)

C [Cell B3] =IF(A3>=60, 1, 0) **D** [Cell B3] =IF(A3>=60, 0, 1)

3 Using spreadsheet software, as shown in the diagram below, 1,000 points were plotted using random numbers between 0 and 1 for the X and Y coordinates. The number of points that fall within the area of a quadrant of a circle with a radius of 1 was examined, and from this, an approximate value of pi was calculated. Answer the following questions.

	A	B	C	D	E	F	G	H
1	**Simulation of approximate calculation of pi**							
2	**Number of times**	**X-coordinate**	**Y-coordinate**	**Distance from the origin**	**Inside-outside determination**		**Probability of being inside a circle**	**Pi**
3	1	0.9090	0.8882	1.6152	0		0.800	3.200
4	2	0.6769	0.1794	0.7003	1			
5	3	0.3127	0.9045	0.9570	1			
6	4	0.4324	0.0900	0.4417	1			
7	5	0.3476	0.8694	0.9363	1			
8	6	0.7042	0.9714	1.1998	0			
9	7	0.1034	0.0875	0.1354	1			

(1) Can this model be represented as a deterministic model or a probabilistic model?

(2) The following formulas are entered in cells C3, E3, G3, and H3 in the diagram. Complete the following formulas by filling in the blanks **A** to **D** with the appropriate answers.

[Cell C3] = **A** ()

[Cell E3] = IF(D3<=1, **B** , 0)

[Cell G3] = **C** (E3:E102)/1000

[Cell H3] = G3* **D**

11-4 Queues

Point!

Queues

([1]**Queue**) : A line formed when customers wait for a service, such as at a supermarket register or an attraction at a theme park.

<Example> Queue at the register of a supermarket

[1] Service start time

・When there are no customers at the register (no queue)

Service start time = Time when a customer ([2]**arrives**) at the register

・When there is a previous customer at the register (when there is a queue)

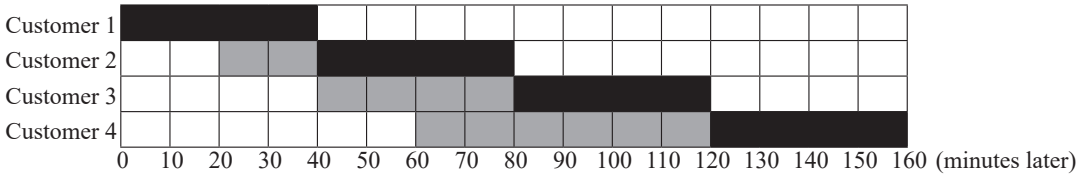
Service start time = Time when the previous customer ([3]**finished**) getting service

[2] Service end time = Service start time + ([4]**Service duration**)

[3] Waiting time = Service start time - ([5]**Arrival time**)

Warm Up

The following diagram represents changes in the queue at a store, with the vertical axis indicating the number of customers in line and the horizontal axis representing the time in minutes since the arrival of customer 1. The time each customer was receiving service is shaded in black, and the time they were waiting in line is shaded in gray. Answer the following questions.



(1) How many minutes did it take to provide service to one customer at this store?

(2) How many minutes after customer 1's arrival was customer 3 able to receive service?

(3) How many minutes did it take from when customer 4 arrived in line until they finished receiving service?

(4) Among customers 1 to 4, who had the longest waiting time?

(5) For how many minutes was the queue the longest?

Explanation

(1) The black cells represent the time when service was being received; therefore, the answer is 40 minutes.

(2) When calculating the time between the arrival of customer 1 and the time customer 3 was able to receive service, 80 minutes have elapsed.

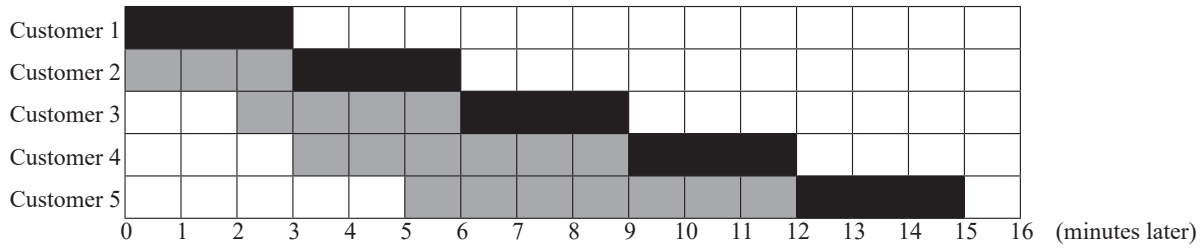
(3) Since customer 4 started waiting in line 60 minutes later and finished receiving service 160 minutes later, the time it took from when customer 4 arrived in line until they finished receiving service is 100 minutes.

(4) From the chart, the waiting times for each customer were as follows: customer 1 was 0 minutes, customer 2 was 20 minutes, customer 3 was 40 minutes, and customer 4 was 60 minutes. Therefore, the customer with the longest waiting time was customer 4.

(5) The queue was longest during the 20-minute period from 60 minutes to 80 minutes, when customers 2, 3, and 4 were waiting in line.

Try

The following diagram represents changes in the line at reception, with the vertical axis indicating the number of customers in line and the horizontal axis representing the time in minutes since the arrival of customer 1. The time each customer was receiving service is shaded in black, and the time they were waiting in line is shaded in gray. Answer the following questions.

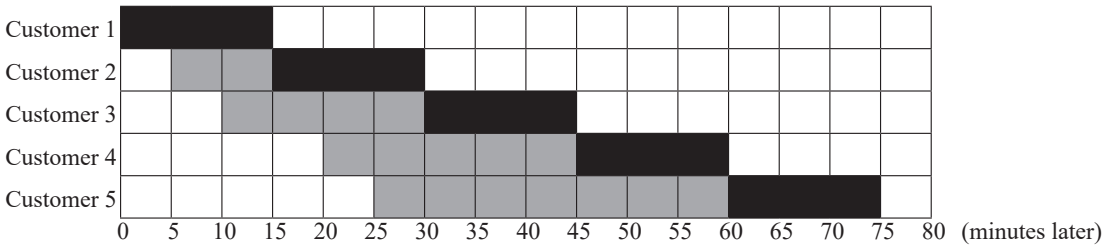


(1) What is the term for a line that forms at reception desks, registers, attractions, etc., where customers line up in order to receive services?
(2) How many minutes was the wait time for customer 2?
(3) How many minutes after the arrival of customer 1 did customer 3 arrive in the queue?
(4) How many minutes after the arrival time of customer 1 did customer 4 finish receiving service?
(5) Among customers 1 to 5, who had the longest waiting time?
(6) What was the maximum length of the queue between the arrival of customer 1 and the completion of service for customer 5?

Exercise

1 Cover the **Point!** section on page 152 with a red sheet and test yourself by writing the items in order in your notebook.

2 The following diagram represents changes in the queue at a store, with the vertical axis indicating the number of customers in line and the horizontal axis representing the time in minutes since the arrival of customer 1. The time each customer was receiving service is shaded in black, and the time they were waiting in line is shaded in gray. Answer the following questions.



(1) How many minutes is the service time per customer at this store?
(2) How many minutes after customer 1's arrival was customer 4 able to receive service?
(3) How many minutes did it take from when customer 5 arrived in line until they finished receiving service?
(4) Among customers 1 to 5, who had the longest waiting time?
(5) For how many minutes was the queue the longest?